

Quantum Econophysics

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The relationships between game theory and quantum mechanics let us propose certain quantization relationships through which we could describe and understand not only quantum but also classical, evolutionary and the biological systems that were described before through the replicator dynamics. Quantum mechanics could be used to explain more correctly biological and economical processes and even it could encloses theories like games and evolutionary dynamics. This could make quantum mechanics a more general theory that we had thought.

Although both systems analyzed are described through two apparently different theories (quantum mechanics and game theory) it is shown that both systems are analogous and thus exactly equivalents. So, we can take some concepts and definitions from quantum mechanics and physics for the best understanding of the behavior of economics and biology. Also, we could maybe understand nature like a game in where its players compete for a common welfare and the equilibrium of the system that they are members.

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I. INTRODUCTION

Could it have an relationship between quantum mechanics and game theories? An actual relationship between these theories that describe two apparently different systems would let us explain biological and economical processes through quantum mechanics, quantum information theory and statistical physics.

We also could try to find a method which let us make quantum a classical system in order to analyze it from a absolutely different perspective and under a physical equilibrium principle which would have to be exactly equivalent to the defined classically in economics or biology. Physics tries to describe approximately nature which is the most perfect system. The equilibrium notion in a physical system is the central cause for this perfection. We could make use of this physical equilibrium to its application in conflictive systems like economics.

The present work analyze the relationships between quantum mechanics and game theory and proposes through certain quantization relationships a quantum understanding of classical systems.

II. THE VON NEUMANN EQUATION & THE STATISTICAL MIXTURE OF STATES

An ensemble is a collection of identically prepared physical systems. When each member of the ensemble is characterized by the same state vector $|\Psi(t)\rangle$ it is called pure ensemble. If each member has a probability p_i of being in the state $|\Psi_i(t)\rangle$ we have a mixed ensemble. Each member of a mixed ensemble is a pure state and its evolution is given by Schrödinger equation. To describe correctly a statistical mixture of states it is necessary the

introduction of the density operator

$$\rho(t) = \sum_{i=1}^n p_i |\Psi_i(t)\rangle \langle \Psi_i(t)| \quad (1)$$

which contains all the physically significant information we can obtain about the ensemble in question. Any two ensembles that produce the same density operator are physically indistinguishable. The density operator can be represented in matrix form. A pure state is specified by $p_i = 1$ for some $|\Psi_i(t)\rangle$, $i = 1, \dots, n$ and the matrix which represents it has all its elements equal to zero except one 1 on the diagonal. The diagonal elements ρ_{nn} of the density operator $\rho(t)$ represents the average probability of finding the system in the state $|n\rangle$ and its sum is equal to 1. The non-diagonal elements ρ_{np} expresses the interference effects between the states $|n\rangle$ and $|p\rangle$ which can appear when the state $|\Psi_i\rangle$ is a coherent linear superposition of these states. Suppose we make a measurement on a mixed ensemble of some observable A . The ensemble average of A is defined by the average of the expected values measured in each member of the ensemble described by $|\Psi_i(t)\rangle$ and with probability p_i , it means $\langle A \rangle_\rho = p_1 \langle A \rangle_1 + p_2 \langle A \rangle_2 + \dots + p_n \langle A \rangle_n$ and can be calculated by using

$$\langle A \rangle = \text{Tr} \{ \rho(t) A \}. \quad (2)$$

The time evolution of the density operator is given by the von Neumann equation

$$i\hbar \frac{d\rho}{dt} = [\hat{H}, \rho] \quad (3)$$

which is only a generalization of the Schrödinger equation and the quantum analogue of Liouville's theorem.

III. THE REPLICATOR DYNAMICS & EGT

Game theory [1, 2, 3] is the study of decision making of competing agents in some conflict situation. It has been applied to solve many problems in economics, social sciences, biology and engineering. The central equilibrium concept in game theory is the Nash Equilibrium which is expressed through the following condition

$$E(p, p) \geq E(r, p). \quad (4)$$

Players are in equilibrium if a change in strategies by any one of them ($p \rightarrow r$) would lead that player to earn less than if he remained with his current strategy (p).

Evolutionary game theory [4, 5, 6] has been applied to the solution of games from a different perspective. Through the replicator dynamics it is possible to solve not only evolutionary but also classical games. That is why EGT has been considered like a generalization of classical game theory. Evolutionary game theory does not rely on rational assumptions but on the idea that the Darwinian process of natural selection [7] drives organisms towards the optimization of reproductive success [8]. Instead of working out the optimal strategy, the different phenotypes in a population are associated with the basic strategies that are shaped by trial and error by a process of natural selection or learning.

The model used in EGT is the following: Each agent in a n -player game where the i^{th} player has as strategy space S_i is modelled by a population of players which have to be partitioned into groups. Individuals in the same group would all play the same strategy. Randomly we make play the members of the subpopulations against each other. The subpopulations that perform the best will grow and those that do not will shrink and eventually will vanish. The process of natural selection assures survival of the best players at the expense of the others. The natural selection process that determines how populations playing specific strategies evolve is known as the replicator dynamics [5, 6, 9, 10]

$$\frac{dx_i}{dt} = [f_i(x) - \langle f(x) \rangle] x_i, \quad (5)$$

$$\frac{dx_i}{dt} = \left[\sum_{j=1}^n a_{ij} x_j - \sum_{k,l=1}^n a_{kl} x_k x_l \right] x_i. \quad (6)$$

The element x_i of the vector x is the probability of playing certain strategy or the relative frequency of individuals using that strategy. The fitness function $f_i = \sum_{j=1}^n a_{ij} x_j$ specifies how successful each subpopulation is, $\langle f(x) \rangle = \sum_{k,l=1}^n a_{kl} x_k x_l$ is the average fitness of the population, and a_{ij} are the elements of the payoff matrix A . The replicator dynamics rewards strategies that outperform the average by increasing their frequency, and penalizes poorly performing strategies by decreasing their

frequency. The stable fixed points of the replicator dynamics are Nash equilibria [2]. If a population reaches a state which is a Nash equilibrium, it will remain there.

The bonestone of EGT is the concept of evolutionary stable strategy (ESS) [4, 11] that is a strengthened notion of Nash equilibrium. It satisfies the following conditions

$$E(p, p) > E(r, p), \\ \text{If } E(p, p) = E(r, p) \text{ then } E(p, r) > E(r, r), \quad (7)$$

where p is the strategy played by the vast majority of the population, and r is the strategy of a mutant present in small frequency. Both p and r can be pure or mixed. An ESS is described as a strategy which has the property that if all the members of a population adopt it, no mutant strategy could invade the population under the influence of natural selection. If a few individuals which play a different strategy are introduced into a population in an ESS, the evolutionary selection process would eventually eliminate the invaders.

IV. RELATIONSHIPS BETWEEN QUANTUM MECHANICS & GAME THEORY

A physical or a socioeconomical system (described through quantum mechanics or game theory) is composed by n members (particles, subsystems, players, states, etc.). Each member is described by a state or a strategy which has assigned a determined probability (x_i or ρ_{ij}). The quantum mechanical system is described by the density operator ρ whose elements represent the system average probability of being in a determined state. In evolutionary game theory the system is defined through a relative frequencies vector x whose elements can represent the frequency of players playing a determined strategy. The evolution of the density operator is described by the von Neumann equation which is a generalization of the Schrödinger equation. While the evolution of the relative frequencies is described through the replicator dynamics (5).

It is important to note that the replicator dynamics is a vectorial differential equation while von Neumann equation can be represented in matrix form. If we would like to compare both systems the first we would have to do is to try to compare their evolution equations by trying to find a matrix representation of the replicator dynamics [12]

$$\frac{dX}{dt} = G + G^T, \quad (8)$$

where the matrix X has as elements

$$x_{ij} = (x_i x_j)^{1/2} \quad (9)$$

and

$$(G + G^T)_{ij} = \frac{1}{2} \sum_{k=1}^n a_{ik} x_k x_{ij} + \frac{1}{2} \sum_{k=1}^n a_{jk} x_k x_{ji} - \sum_{k,l=1}^n a_{kl} x_k x_l x_{ij} \quad (10)$$

are the elements of the matrix $(G + G^T)$.

Although equation (8) is the matrix representation of the replicator dynamics from which we could compare and find a relationship with the von Neumann equation, we can moreover find a Lax representation of the replicator dynamics by calling

$$(G_1)_{ij} = \frac{1}{2} \sum_{k=1}^n a_{ik} x_k x_{ij}, \quad (11)$$

$$(G_2)_{ij} = \frac{1}{2} \sum_{k=1}^n a_{jk} x_k x_{ji}, \quad (12)$$

$$(G_3)_{ij} = \sum_{k,l=1}^n a_{kl} x_k x_l x_{ij} \quad (13)$$

the elements of the matrixes G_1 , G_2 and G_3 that compose by adding the matrix $(G + G^T)$. The matrixes G_1 , G_2 and G_3 can be also factorized in function of the matrixes Q and X

$$G_1 = QX, \quad (14)$$

$$G_2 = XQ, \quad (15)$$

$$G_3 = 2XQX, \quad (16)$$

where Q is a diagonal matrix and has as elements $q_{ii} = \frac{1}{2} \sum_{k=1}^n a_{ik} x_k$. By using the fact that $X^2 = X$ we can write the equation (8) like

$$\frac{dX}{dt} = QXX + XXQ - 2XQX \quad (17)$$

and finally, by grouping into commutators and defining $\Lambda = [Q, X]$

$$\frac{dX}{dt} = [\Lambda, X]. \quad (18)$$

The matrix Λ has as elements

$$(\Lambda)_{ij} = \frac{1}{2} \left[\left(\sum_{k=1}^n a_{ik} x_k \right) x_{ij} - x_{ji} \left(\sum_{k=1}^n a_{jk} x_k \right) \right].$$

This matrix commutative form of the replicator dynamics (18) follows the same dynamic as the von Neumann equation (3) and the properties of their correspondent elements (matrixes) are similar, being the properties corresponding to our quantum system more general than the properties of the classical system.

The next table shows some specific resemblances between quantum statistical mechanics and evolutionary game theory [13].

Table 1

Quantum Statistical Mechanics	Evolutionary Game Theory
n system members	n population members
Each member in the state $ \Psi_k\rangle$	Each member plays strategy s_i
$ \Psi_k\rangle$ with $p_k \rightarrow \rho_{ij}$	$s_i \rightarrow x_i$
$\rho, \sum_i \rho_{ii} = 1$	$X, \sum_i x_i = 1$
$i\hbar \frac{d\rho}{dt} = [\hat{H}, \rho]$	$\frac{dX}{dt} = [\Lambda, X]$
$S = -Tr\{\rho \ln \rho\}$	$H = -\sum_i x_i \ln x_i$

In table 2 we show the properties of the matrixes ρ and X .

Table 2

Density Operator	Relative freq. Matrix
ρ is Hermitian	X is Hermitian
$Tr\rho(t) = 1$	$TrX = 1$
$\rho^2(t) \leq \rho(t)$	$X^2 = X$
$Tr\rho^2(t) \leq 1$	$TrX^2(t) = 1$

Although both systems are different, both are analogous and thus exactly equivalents.

V. QUANTUM REPLICATOR DYNAMICS & THE QUANTIZATION RELATIONSHIPS

The resemblances between both systems and the similarity in the properties of their corresponding elements let us to define and propose the next quantization relationships

$$x_i \rightarrow \sum_{k=1}^n \langle i | \Psi_k \rangle p_k \langle \Psi_k | i \rangle = \rho_{ii},$$

$$(x_i x_j)^{1/2} \rightarrow \sum_{k=1}^n \langle i | \Psi_k \rangle p_k \langle \Psi_k | j \rangle = \rho_{ij}. \quad (19)$$

A population will be represented by a quantum system in which each subpopulation playing strategy s_i will be represented by a pure ensemble in the state $|\Psi_k(t)\rangle$ and with probability p_k . The probability x_i of playing strategy s_i or the relative frequency of the individuals using strategy s_i in that population will be represented as the probability ρ_{ii} of finding each pure ensemble in the state $|i\rangle$ [12].

Through these quantization relationships the replicator dynamics (in matrix commutative form) (18) takes the form of the equation of evolution of mixed states (3). And also

$$X \longrightarrow \rho, \quad (20)$$

$$\Lambda \longrightarrow -\frac{i}{\hbar} \hat{H}, \quad (21)$$

where \hat{H} is the Hamiltonian of the physical system.

The equation of evolution of mixed states from quantum statistical mechanics (3) is the quantum analogue of the replicator dynamics in matrix commutative form (18).

VI. GAMES THROUGH STATISTICAL MECHANICS & QIT

There exists a strong relationship between game theories, statistical mechanics and information theory. The bonds between these theories are the density operator and entropy [14, 15]. From the density operator we can construct and understand the statistical behavior about our system by using the statistical mechanics. Also we can develop the system in function of its accessible information and analyze it through information theories under a criterion of maximum or minimum entropy.

Entropy is the central concept of information theories [16, 17]. The Shannon entropy expresses the average information we expect to gain on performing a probabilistic experiment of a random variable A which takes the values a_i with the respective probabilities p_i . It also can be seen as a measure of uncertainty before we learn the value of A . We define the Shannon entropy of a random variable A by

$$H(A) \equiv H(p_1, \dots, p_n) \equiv - \sum_{i=1}^n p_i \log_2 p_i. \quad (22)$$

The entropy of a random variable is completely determined by the probabilities of the different possible values that the random variable takes. Due to the fact that $p = (p_1, \dots, p_n)$ is a probability distribution, it must satisfy $\sum_{i=1}^n p_i = 1$ and $0 \leq p_1, \dots, p_n \leq 1$. The Shannon entropy of the probability distribution associated with the source gives the minimal number of bits that are needed in order to store the information produced by a source, in the sense that the produced string can later be recovered.

The von Neumann entropy [16, 17] is the quantum analogue of Shannon's entropy but it appeared 21 years before and generalizes Boltzmann's expression. Entropy in quantum information theory plays prominent roles in many contexts, e.g., in studies of the classical capacity of a quantum channel [18, 19] and the compressibility of a quantum source [20, 21]. Quantum information theory appears to be the basis for a proper understanding of the emerging fields of quantum computation [22, 23], quantum communication [24, 25], and quantum cryptography [26, 27].

Suppose A and B are two random variables. The joint entropy $H(A, B)$ measures our total uncertainty about

the pair (A, B) and it is defined by

$$H(A, B) \equiv - \sum_{i,j} p_{ij} \log_2 p_{ij} \quad (23)$$

while

$$H(A) = - \sum_{i,j} p_{ij} \log_2 \sum_j p_{ij}, \quad (24)$$

$$H(B) = - \sum_{i,j} p_{ij} \log_2 \sum_i p_{ij}, \quad (25)$$

where p_{ij} is the joint probability to find A in state a_i and B in state b_j .

The conditional entropy $H(A | B)$ is a measure of how uncertain we are about the value of A , given that we know the value of B . The entropy of A conditional on knowing that B takes the value b_j is defined by

$$\begin{aligned} H(A | B) &\equiv H(A, B) - H(B), \\ H(A | B) &\equiv - \sum_{i,j} p_{ij} \log_2 p_{i|j}, \end{aligned} \quad (26)$$

where $p_{i|j} = \frac{p_{ij}}{\sum_i p_{ij}}$ is the conditional probability that A is in state a_i given that B is in state b_j .

The mutual or correlation entropy $H(A : B)$ measures how much information A and B have in common. The mutual or correlation entropy $H(A : B)$ is defined by

$$\begin{aligned} H(A : B) &\equiv H(A) + H(B) - H(A, B), \\ H(A : B) &\equiv - \sum_{i,j} p_{ij} \log_2 p_{i:j}, \end{aligned} \quad (27)$$

where $p_{i:j} = \frac{\sum_i p_{ij} \sum_j p_{ij}}{p_{ij}}$ is the mutual probability. The mutual or correlation entropy also can be expressed through the conditional entropy via

$$H(A : B) = H(A) - H(A | B), \quad (28)$$

$$H(A : B) = H(B) - H(B | A). \quad (29)$$

The joint entropy would equal the sum of each of A 's and B 's entropies only in the case that there are no correlations between A 's and B 's states. In that case, the mutual entropy or information vanishes and we could not make any predictions about A just from knowing something about B .

The relative entropy $H(p || q)$ measures the closeness of two probability distributions, p and q , defined over the same random variable A . We define the relative entropy of p with respect to q by

$$\begin{aligned} H(p || q) &\equiv \sum_i p_i \log_2 p_i - \sum_i p_i \log_2 q_i, \\ H(p || q) &\equiv -H(A) - \sum_i p_i \log_2 q_i. \end{aligned} \quad (30)$$

The relative entropy is non-negative, $H(p \parallel q) \geq 0$, with equality if and only if $p = q$. The classical relative entropy of two probability distributions is related to the probability of distinguishing the two distributions after a large but finite number of independent samples (Sanov's theorem) [28].

By analogy with the Shannon entropies it is possible to define conditional, mutual and relative quantum entropies. Quantum entropies also satisfies many other interesting properties that do not satisfy their classical analogues. For example, the conditional entropy can be negative and its negativity always indicates that two systems are entangled and indeed, how negative the conditional entropy is provides a lower bound on how entangled the two systems are [29].

By other hand, in statistical mechanics entropy can be regarded as a quantitative measure of disorder. It takes its maximum possible value in a completely random ensemble in which all quantum mechanical states are equally likely and is equal to zero in the case of a pure ensemble which has a maximum amount of order because all members are characterized by the same quantum mechanical state ket.

From both possible points of view and analysis (statistical mechanics or information theories) of the same system its entropy is exactly the same. Lets consider a system composed by N members, players, strategies, states, etc. This system is described completely through certain density operator ρ , its evolution equation (the von Neumann equation) and its entropy. Classically, the system is described through the matrix of relative frequencies X , the replicator dynamics and the Shannon entropy. For the quantum case we define the von Neumann entropy as

$$S = -Tr \{ \rho \ln \rho \} \quad (31)$$

and for the classical case

$$H = - \sum_{i=1} x_{ii} \ln x_{ii} \quad (32)$$

which is the Shannon entropy over the relative frequencies vector x (the diagonal elements of X).

We can describe the evolution of the entropy of our classical system $H(t)$ by supposing that the vector of relative frequencies $x(t)$ evolves in time following the replicator dynamics [15]

$$\frac{dH}{dt} = Tr \left\{ U(\tilde{H} - X) \right\}, \quad (33)$$

where \tilde{H} is a diagonal matrix whose trace is equal to the Shannon entropy i.e. $H = Tr \tilde{H}$ and $U_i = [f_i(x) - \langle f(x) \rangle]$.

In a far from equilibrium system the von Neumann vary in time until it reaches its maximum value. When the dynamics is chaotic the variation with time of the physical entropy goes through three successive, roughly separated stages [30]. In the first one, $S(t)$ is dependent on the details of the dynamical system and of the initial distribution, and no generic statement can be made. In the second stage, $S(t)$ is a linear increasing function of time ($\frac{dS}{dt} = const.$). In the third stage, $S(t)$ tends asymptotically towards the constant value which characterizes equilibrium ($\frac{dS}{dt} = 0$). With the purpose of calculating the time evolution of entropy we approximate the logarithm of ρ by series $\ln \rho = (\rho - I) - \frac{1}{2}(\rho - I)^2 + \frac{1}{3}(\rho - I)^3 \dots$ [15]

$$\begin{aligned} \frac{dS(t)}{dt} = & \frac{11}{6} \sum_i \frac{d\rho_{ii}}{dt} \\ & - 6 \sum_{i,j} \rho_{ij} \frac{d\rho_{ji}}{dt} \\ & + \frac{9}{2} \sum_{i,j,k} \rho_{ij} \rho_{jk} \frac{d\rho_{ki}}{dt} \\ & - \frac{4}{3} \sum_{i,j,k,l} \rho_{ij} \rho_{jk} \rho_{kl} \frac{d\rho_{li}}{dt} + \zeta. \end{aligned} \quad (34)$$

In general entropy can be maximized subject to different constrains. In each case the result is the condition the system must follow to maximize its entropy. Generally, this condition is a probability distribution function. We can obtain the density operator from the study of an ensemble in thermal equilibrium. Nature tends to maximize entropy subject to the constraint that the ensemble average of the Hamiltonian has a certain prescribed value. We will maximize S by requiring that

$$\delta S = - \sum_i \delta \rho_{ii} (\ln \rho_{ii} + 1) = 0 \quad (35)$$

subject to the constrains $\delta Tr(\rho) = 0$ and $\delta \langle E \rangle = 0$. By using Lagrange multipliers

$$\sum_i \delta \rho_{ii} (\ln \rho_{ii} + \beta E_i + \gamma + 1) = 0 \quad (36)$$

and the normalization condition $Tr(\rho) = 1$ we find that

$$\rho_{ii} = \frac{e^{-\beta E_i}}{\sum_k e^{-\beta E_k}} \quad (37)$$

which is the condition that the density operator and its elements must satisfy to our system tends to maximize its entropy S . If we maximize S without the internal energy constrain $\delta \langle E \rangle = 0$ we obtain

$$\rho_{ii} = \frac{1}{N} \quad (38)$$

which is the $\beta \rightarrow 0$ limit ("high - temperature limit") in equation (37) in where a canonical ensemble becomes a

completely random ensemble in which all energy eigenstates are equally populated. In the opposite low - temperature limit $\beta \rightarrow \infty$ tell us that a canonical ensemble becomes a pure ensemble where only the ground state is populated. The parameter β is related to the “temperature” τ as follows

$$\beta = \frac{1}{\tau}. \quad (39)$$

By replacing ρ_{ii} obtained in the equation (37) in the von Neumann entropy we can rewrite it in function of the partition function $Z = \sum_k e^{-\beta E_k}$, β and $\langle E \rangle$ through the next equation

$$S = \ln Z + \beta \langle E \rangle. \quad (40)$$

From the partition function we can know some parameters that define the system like

$$\langle E \rangle = -\frac{1}{Z} \frac{\partial Z}{\partial \beta} = -\frac{\partial \ln Z}{\partial \beta}, \quad (41)$$

$$\langle \Delta E^2 \rangle = -\frac{\partial \langle E \rangle}{\partial \beta} = -\frac{1}{\beta} \frac{\partial S}{\partial \beta}. \quad (42)$$

We can also analyze the variation of entropy with respect to the average energy of the system

$$\frac{\partial S}{\partial \langle E \rangle} = \frac{1}{\tau}, \quad (43)$$

$$\frac{\partial^2 S}{\partial \langle E \rangle^2} = -\frac{1}{\tau^2} \frac{\partial \tau}{\partial \langle E \rangle} \quad (44)$$

and with respect to the parameter β

$$\frac{\partial S}{\partial \beta} = -\beta \langle \Delta E^2 \rangle, \quad (45)$$

$$\frac{\partial^2 S}{\partial \beta^2} = \frac{\partial \langle E \rangle}{\partial \beta} + \beta \frac{\partial^2 \langle E \rangle}{\partial \beta^2}. \quad (46)$$

VII. FROM CLASSICAL TO QUANTUM

The resemblances between both systems (described through quantum mechanics and EGT) apparently different but analogous and thus exactly equivalents and the similarity in the properties of their corresponding elements let us to define and propose the quantization relationships like in section 5.

It is important to note that equation (18) is nonlinear while its quantum analogue is linear. This means that the quantization eliminates the nonlinearities. Also through this quantization the classical system that were described through a diagonal matrix X can be now described through a density operator which not necessarily must describe a pure state, i.e. its non diagonal elements can be different from zero representing a mixed state due to the coherence between quantum states that were not present through a classical analysis.

Through the relationships between both systems we could describe classical, evolutionary, quantum and also the biological systems that were described before through evolutionary dynamics with the replicator dynamics. We could explain through quantum mechanics biological and economical processes being a much more general theory that we had thought. It could even encloses theories like games and evolutionary dynamics.

Problems in economy and finance have attracted the interest of statistical physicists. Kobelev et al [31] used methods of statistical physics of open systems for describing the time dependence of economic characteristics (income, profit, cost, supply, currency, etc.) and their correlations with each other. Antoniou et al [32] introduced a new approach for the presentation of economic systems with a small number of components as a statistical system described by density functions and entropy. This analysis is based on a Lorenz diagram and its interpolation by a continuous function. Conservation of entropy in time may indicate the absence of macroscopic changes in redistribution of resources. Assuming the absence of macro-changes in economic systems and in related additional expenses of resources, we may consider the entropy as an indicator of efficiency of the resources distribution. Statistical physicists are also extremely interested in economic fluctuations [33] in order to help our world financial system avoid “economic earthquakes”. Also it is suggested that in the field of turbulence, we may find some crossover with certain aspects of financial markets. Statistical mechanics and economics study big ensembles: collections of atoms or economic agents, respectively. The fundamental law of equilibrium statistical mechanics is the Boltzmann-Gibbs law, which states that the probability distribution of energy E is $P(E) = C e^{-E/T}$, where T is the temperature, and C is a normalizing constant. The main ingredient that is essential for the derivation of the Boltzmann-Gibbs law is the conservation of energy. Thus, one may generalize that any conserved quantity in a big statistical system should have an exponential probability distribution in equilibrium [34]. In a closed economic system, money is conserved. Thus, by analogy with energy, the equilibrium probability distribution of money must follow the exponential Boltzmann-Gibbs law characterized by an effective temperature equal to the average amount of money per economic agent. Drăgulescu and Yakovenko demonstrated how the Boltzmann-Gibbs distribution emerges in computer simulations of economic models. They considered a thermal machine, in which the difference of temperature allows one to extract a monetary profit. They also discussed the role of debt, and models with broken time-reversal symmetry for which the Boltzmann-Gibbs law does not hold. Recently the insurance market, which is one of the important branches of economy, have attracted the attention of physicists [35]. The maximum entropy principle is used for pricing the insurance. Darrooneh obtained the price density based on this principle,

applied it to multi agents model of insurance market and derived the utility function. The main assumption in his work is the correspondence between the concept of the equilibrium in physics and economics. He proved that economic equilibrium can be viewed as an asymptotic approximation to physical equilibrium and some difficulties with mechanical picture of the equilibrium may be improved by considering the statistical description of it. Topsøe [36] also has suggested that thermodynamical equilibrium equals game theoretical equilibrium. Quantum games have proposed a new point of view for the solution of the classical problems and dilemmas in game theory. Quantum games are more efficient than classical games and provide a saturated upper bound for this efficiency [37, 38, 39, 40, 41, 42].

Nature may be playing quantum survival games at the molecular level [43, 44]. It could lead us to describe many of the life processes through quantum mechanics like Gogonea and Merz [45] who indicated that games are being played at the quantum mechanical level in protein folding. Gafiychuk and Prykarpatsky [46] applied the replicator equations written in the form of nonlinear von Neumann equations to the study of the general properties of the quasispecies dynamical system from the standpoint of its evolution and stability. They developed a mathematical model of a naturally fitted coevolving ecosystem and a theoretical study a self-organization problem of an ensemble of interacting species. The genetic code is the relationship between the sequence of the bases in the DNA and the sequence of amino acids in proteins. Recent work [47] about evolvability of the genetic code suggests that the code is shaped by natural selection. DNA is a nonlinear dynamical system and its evolution is a sequence of chemical reactions. An abstract DNA-type system is defined by a set of nonlinear kinetic equations with polynomial nonlinearities that admit soliton solutions associated with helical geometry. Aerts and Czachor [48] shown that the set of these equations allows for two different Lax representations: They can be written as von Neumann type nonlinear systems and they can be regarded as a compatibility condition for a Darboux-covariant Lax pair. Organisms whose DNA evolves in a chaotic way would be eliminated by natural selection. They also explained why non-Kolmogorovian probability models occurring in soliton kinetics are naturally associated with chemical reactions. Patel [49, 50] suggested quantum dynamics played a role in the DNA replication and the optimization criteria involved in genetic information processing. He considers the criteria involved as a task similar to an unsorted assembly operation where the Grover's database search algorithm fruitfully applies; given the different optimal solutions for classical and quantum dynamics. Turner and Chao [51] studied the evolution of competitive interactions among viruses in an RNA phage, and found that the fitness of the phage generates a payoff matrix conforming to the two-person prisoner's dilemma game. Bacterial infections by viruses have been presented

as classical game-like situations where nature prefers the dominant strategies. Azhar Iqbal [42] showed results in which quantum mechanics has strong and important roles in selection of stable solutions in a system of interacting entities. These entities can do quantum actions on quantum states. It may simply consists of a collection of molecules and the stability of solutions or equilibria can be affected by quantum interactions which provides a new approach towards theories of rise of complexity in groups of quantum interacting entities. Neuroeconomics [52, 53] may provide an alternative to the classical Cartesian model of the brain and behavior [54] through a rich dialogue between theoretical neurobiology and quantum logic [55, 56].

The results shown in this study on the relationships between quantum mechanics and game theories are a reason of the applicability of physics in economics and biology. Both systems described through two apparently different theories are analogous and thus exactly equivalents. So, we can take some concepts and definitions from quantum mechanics and physics for the best understanding of the behavior of economics and biology. Also, we could maybe understand nature like a game in where its players compete for a common welfare and the equilibrium of the system that they are members.

VIII. ON A QUANTUM UNDERSTANDING OF CLASSICAL SYSTEMS

If our systems are analogous and thus exactly equivalents, our physical equilibrium (maximum entropy) should be also exactly equivalent to our socioeconomical equilibrium. If in an isolated system each of its accessible states do not have the same probability, the system is not in equilibrium. The system will vary and will evolution in time until it reaches the equilibrium state in where the probability of finding the system in each of the accessible states is the same. The system will find its more probable configuration in which the number of accessible states is maximum and equally probable. The whole system will vary and rearrange its state and the states of its ensembles with the purpose of maximize its entropy and reach its maximum entropy state. We could say that the purpose and maximum payoff of a physical system is its maximum entropy state. The system and its members will vary and rearrange themselves to reach the best possible state for each of them which is also the best possible state for the whole system.

This can be seen like a microscopical cooperation between quantum objects to improve their states with the purpose of reaching or maintaining the equilibrium of the system. All the members of our quantum system will play a game in which its maximum payoff is the equilibrium of the system. The members of the system act as a whole besides individuals like they obey a rule in where they

prefer the welfare of the collective over the welfare of the individual. This equilibrium is represented in the maximum system entropy in where the system resources are fairly distributed over its members. A system is stable only if it maximizes the welfare of the collective above the welfare of the individual. If it is maximized the welfare of the individual above the welfare of the collective the system gets unstable and eventually it collapses (Collective Welfare Principle [12, 13, 15]).

Fundamentally, we could distinguish three states in every system: minimum entropy, maximum entropy, and when the system is tending to whatever of these two states. The natural trend of a physical system is to the maximum entropy state. The minimum entropy state is a characteristic of a manipulated system i.e. externally controlled or imposed. A system can be internally or externally manipulated or controlled with the purpose of guide it to a state of maximum or minimum entropy depending of the ambitions of the members that compose it or the people who control it.

There exists tacit rules inside a system. These rules do not need to be specified or clarified and search the system equilibrium under the collective welfare principle. The other prohibitive and repressive rules are imposed over the system when one or many of its members violate the collective welfare principle and search to maximize its individual welfare at the expense of the group. Then it is necessary to establish regulations on the system to try to reestablish the broken natural order.

IX. CONCLUSIONS

The relationships between game theory and quantum mechanics let us propose certain quantization relationships through which we could describe and understand not only classical and evolutionary systems but also the biological systems that were described before through the replicator dynamics. Quantum mechanics could be used

to explain more correctly biological and economical processes and even encloses theories like games and evolutionary dynamics.

The quantum analogues of the relative frequencies matrix, the replicator dynamics and the Shannon entropy are the density operator, the von Neumann equation and the von Neumann entropy. Every game (classical, evolutionary or quantum) can be described quantically through these three elements.

The bonds between game theories, statistical mechanics and information theory are the density operator and entropy. From the density operator we can construct and obtain all the mechanical statistical information about our system. Also we can develop the system in function of its information and analyze it through information theories under a criterion of maximum or minimum entropy.

Although both systems analyzed are described through two apparently different theories (quantum mechanics and game theory) both are analogous and thus exactly equivalents. So, we can take some concepts and definitions from quantum mechanics and physics for the best understanding of the behavior of economics and biology. Also, we could maybe understand nature like a game in where its players compete for a common welfare and the equilibrium of the system that they are members.

We could say that the purpose and maximum payoff of a system is its maximum entropy state. The system and its members will vary and rearrange themselves to reach the best possible state for each of them which is also the best possible state for the whole system. This can be seen like a microscopical cooperation between quantum objects to improve their states with the purpose of reaching or maintaining the equilibrium of the system. All the members of our system will play a game in which its maximum payoff is the equilibrium of the system. The members of the system act as a whole besides individuals like they obey a rule in where they prefer to work for the welfare of the collective besides the individual welfare.

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